

## Appendix A

### Drude-model of graphene

In This calculation, we used the Drude-like intraband contribution model of surface conductivity of graphene which is states in [12].

Inside a simple source free medium, where the medium is not conduct, we have  $\rho=0$ ,  $\vec{j}=0$  and  $\sigma=0$ , so the time harmonic Maxwell's equations are:

$$\nabla \times \vec{E} = -j\omega\mu\vec{H} \quad (\text{A.1})$$

$$\nabla \times \vec{H} = j\omega\varepsilon\vec{E} \quad (\text{A.2})$$

$$\nabla \cdot \vec{E} = 0 \quad (\text{A.3})$$

$$\nabla \cdot \vec{H} = 0 \quad (\text{A.4})$$

If  $\sigma \neq 0$ , it means that we have a conducting media, the equation (2) will be:

$$\nabla \times \vec{H} = \vec{J} + j\omega\varepsilon\vec{E} = (\sigma + j\omega\varepsilon)\vec{E} = j\omega\left(\varepsilon + \frac{\sigma}{j\omega}\right)\vec{E} = j\omega\varepsilon_c\vec{E} \quad (\text{A.5})$$

Where  $\vec{J} = \sigma\vec{E}$  and  $\varepsilon_c = j\frac{\sigma}{\omega}$  as complex permittivity. Also, we can use the permittivity of Drude formula:

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - j\gamma)} \quad (\text{A.6})$$

Now, we have one permittivity from Maxwell's equation and another from Drude model which we can equal two permittivity as below:

$$\varepsilon_r(\omega) = 1 - \frac{\omega_p^2}{\omega(\omega - j\gamma)} = 1 - j\frac{\sigma(\omega)}{\varepsilon_0\omega} = 1 - j\frac{\sigma_{surf}(\omega)}{\omega\varepsilon_0\Delta} \quad (\text{A.7})$$

Where  $\sigma = \frac{\sigma(\omega)}{\Delta}$  and  $\Delta$  is the thickness of graphene plate.

With simplifying the two sides of the equation:

$$\frac{\omega_p^2}{(\omega - j\gamma)} = j\frac{\sigma_{surf}(\omega)}{\varepsilon_0\Delta} \quad (\text{A.8})$$

The Kremers et al in their scientific article [12] states that the surface conductivity for an infinite graphene film can be calculated by the Kubo formalism. Since our structure is finite one, random phase approximation can be used and within this the conductivity in a local form with a Drude-like intraband contribution can be represented as below:

$$\sigma(\omega, \mu_c, \tau, T) = \frac{2e^2 k_B T}{\pi \hbar^2} \ln \left[ 2 \cosh \left[ \frac{\mu_c}{2k_B T} \right] \right] \frac{-j}{\omega - j\tau^{-1}} \quad (\text{A.9})$$

By substituting equation (A.9) in equation (A.8), we have:

$$\frac{\omega_p^2}{(\omega - j\gamma)} = \frac{j}{\varepsilon_0 \Delta} \frac{2e^2}{\pi \hbar} \frac{k_B T}{\hbar} \ln \left[ 2 \cosh \left[ \frac{\mu_c}{2k_B T} \right] \right] \frac{-j}{\omega - j\tau^{-1}}, \quad \gamma = \tau^{-1} = 10^{-13} \text{ s}^{-1} \quad (\text{A.10})$$

And equation (A.10) can be reduced to:

$$\omega_p = \sqrt{\frac{2e^2}{\varepsilon_0 \Delta \pi \hbar} \frac{k_B T}{\hbar} \ln \left[ 2 \cosh \left[ \frac{\mu_c}{2k_B T} \right] \right]} \quad (\text{A.11})$$

Where  $e=1.602 \times 10^{-19}$  is the charge of an electron and the variable  $\mu_c$  is the Chemical potential.